

Multiscale and directional transform for denoising Medical Images

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ABSTRACT

In medical images, noise suppression is a particularly delicate and difficult task. A trade off between noise reduction and the preservation of actual image features has to be made in a way that enhances the diagnostically relevant image content. The main properties of a good image denoising model are that it will remove noise while preserving edges and contours. This paper proposes a new denoising technique called Contourlet transform. This method of contourlets has been used extensively for denoising medical image. It provides a flexible multi resolution, local and directional image expansion. The contourlet transform is realized efficiently via double iterated filter bank structure where the Laplacian Pyramid (LP) filter in the first stage is used to capture the point discontinuities, and then followed by a directional filter bank (DFB) to link the point discontinuities.

In this paper, we evaluate Contourlets denoising procedures using medical test images corrupted with additive Gaussian noise, salt&pepper noise, multiplicative noise and compare the performance with FBP, Wavelets, in terms of the peak-signal-to-noise ratio (PSNR) as a measure of the quality of denoising. Experimental results shows that the contourlet transform outperforms the other two techniques.

Keywords— Contourlets, Denoising, FBP, Laplacian Pyramid (LP), Tomography, Wavelets.

I.INTRODUCTION

Interest in digital image processing methods stems from two principal application areas: improvement of pictorial information for human interpretation and processing of image data for storage, transmission, and representation for autonomous machine perception. An image is often corrupted by noise in its acquisition and transmission. Image denoising is used to remove the noise while retaining as much as possible the important signal features. All denoising methods show an outstanding performance when the image models corresponds to the algorithm assumptions, but fails in general and create artifacts or remove image fine structures. In the filtered back projection technique We are interested in the recovery of an image f from its tomographic projections Y , also called sinograms, and defined as:

$$Y = R f + W \quad (1)$$

Where $f[n_1, n_2] \in C^{N^2}$ is the observed image, W is an additive noise, usually modelled as Poisson or Gaussian noise, and R is the discrete Radon transform. The discrete Radon transform is derived from its continuous version R_c , which is equivalent to the X-ray transform in two dimensions and is defined as

$$P_\theta(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y) \delta(x \cos \theta + y \sin \theta - t) dx dy \quad (2)$$

Where $f(x, y) \in \Lambda^2$, δ is the Dirac delta function and $\theta \in [0, 2\pi]$. In the discrete Radon transform, a line integral along $x \cos \theta + y \sin \theta = t$ can be approximated by a summation of the pixel values inside the strip $t - \frac{1}{2} < n_1 \cos \theta + n_2 \sin \theta < t + \frac{1}{2}$. The generation of sinogram data corrupted by additive Gaussian noise is shown in Figure.1.

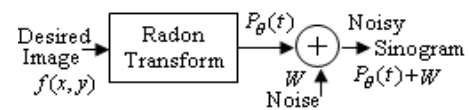


Fig.1 Generation of Noisy Sinogram

Computerized Tomography measure the density or the metabolic activity of a section of the patient's body (i.e., roughly speaking, produce sinograms Y), and an estimation of the image $f(x, y)$ representing the observed section is derived by a tomographic reconstruction procedure from the sinograms. This sinogram is filtered by using a ramp filter to eliminate the noise and then inverse radon transform is obtained which is also called as back projection to reconstruct the original image. The back projection technique suffers from blurring operator. FBP suffers from serious performance limitations, due to the fact that the vectors of the Fourier basis provide a good representation (diagonalization) of the Radon operator, but are not adapted to represent spatially inhomogeneous data such as medical images. A fundamental difficulty of tomographic reconstruction comes from the fact that the Radon transform is a smoothing transform, and inverting the Radon transform in presence of additive noise is an ill-posed inverse problem, because R^{-1} is not a bounded linear operator; numerically speaking, a direct computation of $R^{-1}f$ would be contaminated by a huge additive noise $z = R^{-1}W$. [1].

To improve the performance of tomographic reconstruction procedures, researchers have studied iterative statistical model based techniques. These approaches can provide a significant improvement over FBP. One of such

technique is the Wavelet transform to estimate the signal from the noisy data. For, 1-D piecewise smooth signals like scan line of an image, 1-D wavelets have been established as the right tool. However natural images are not simple stacks of, 1-D piecewise smooth scan lines; discontinuity points that is edges are typically located along smooth curves that is contours owing to smooth boundaries of physical objects. As a result of separable extension from 1-D bases wavelets in 2-D are good at isolated the discontinuities at edge points, but will not see smoothness along contours[4]. The weakness of the wavelet domain is that it typically does not economically represent the noise. Therefore the wavelet shrinkage is unsatisfactory with large MSE; the estimate is either noisy or distorted. The limitations of commonly used separable extensions of 1-D transforms, such as Fourier and Wavelet transforms in capturing the geometry of image edges are well known.

In this paper we perceive a true 2-D transform called contourlet transform that can capture the intrinsic geometrical structure that is key in visual information [2]. This method of contourlets has been used extensively for denoising medical image. It provides a flexible multi resolution, local and directional image expansion. The contourlet transform is realized efficiently via double iterated filter bank structure where the Laplacian Pyramid (LP) filter in the first stage is used to capture the point discontinuities, and then followed by a directional filter bank (DFB) to link the point discontinuities[9].

II.LIMITATIONS OF OTHER DENOISING TECHNIQUES

Filtered back projection (FBP) method of image denoising uses inverse Radon transform. But the inversion of the radon transform in the presence of noise is numerically unstable in tomographic image reconstruction and is said to be ill conditioned. Wavelet transform, a multiscale transform is good at isolating the discontinuities at edge points, but will not see the smoothness along the contours.

The image denoising techniques such as the steerable pyramid, brushlets which are multiscale and directional image representations do not allow for a different number of directions at each scale while achieving nearly critical sampling. The recently developed approaches for the efficient representations of geometrical regularity including the image denoising techniques such as ridgelets, wedgelets, require an edge-detection stage, followed by an adaptive representation [8]. By contrast, curvelet representation is fixed transform. This feature allows it to be easily applied in a wide range of image processing tasks, similar to wavelets. The curvelet construction simple in the continuous domain but causes the implementation for discrete images – sampled on a rectangular grid – to be very challenging. In particular, approaching critical sampling seems difficult in such discretized constructions [6].

III.IMAGE DENOISING USING CONTOURLETS

The good image denoising technique should possess the following 5 properties

1. Multiresolution
2. Localization
3. Critical sampling
4. Directionality
5. Anisotropy

Among these desired data the first three are successfully provided by separable wavelets, while the last two are provided by the contourlet transform. This contourlets [9] uses nonseparable double filtered structure for obtaining the sparse expansions for typical images having smooth contours. In this double filter bank the laplacian pyramid is first used to capture the points discontinuities (multiresolution expansion) and then followed by directional filter banks to link point discontinuities(multi directional expansion).

III.I Review of contourlet Transform

Pyramid frames

One way to obtain a multiscale decomposition is to use Laplacian pyramid (LP). This LP decomposition at each level generates a down sampled low pass version of the original image and the prediction resulting in a band pass image. The following figure depicts this decomposition process where H and G are called (low pass) Analysis and synthesis filters, respectively and M is sampling matrix. The process can be iterated on the course (down sampled low pass) signal. In multidimensional filter banks, sampling is represented by sampling matrices; for example, down sampling $x[n]$ by M yields $x_d[n] = x[Mn]$, where M is an integer matrix.

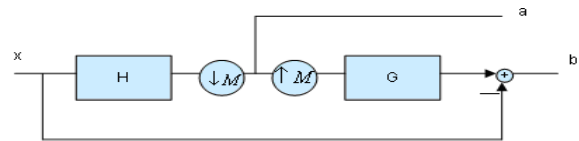


Fig:2 LPone level of decomposition

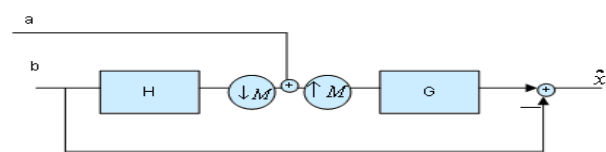


Fig:3 The new reconstruction scheme for the LP

The LP has distinguishing feature that this does not have scrambled frequencies. This frequency scrambling happens in the wavelets filter bank when a high pass channel, after down sampling is folded back into low frequency band and thus its spectrum is reflected. In the LP, this effect is avoided by down sampling the low pass channel only.

The LP orthogonal filters that is analysis and synthesis filters, or time reversal, $h[n]=g[-n]$ and $g[n]$ is orthogonal to its translates with respect to sampling lattice by M provides a tight frame with frame bounds are equal to one. In this case, we proposed the use of optimal linear reconstruction using dual frame operator as shown in Fig.3. The new reconstruction differs from the usual method where the signal is obtained by simply adding back the difference to prediction from coarse signal and was shown

to achieve significant improvement over usual reconstruction in presence of noise .

Iterated directional filter banks:

The directional filterbanks (DFB) is efficiently implemented via an l-level binary tree decomposition that leads to 2^l sub bands with wedge shaped frequency partitioning as shown in Fig.4

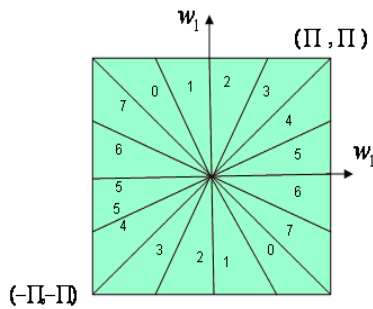


Fig:4

Directional filter bank frequency partitioning where $l = 3$ and there are $2^3 = 8$ real wedge-shaped frequency bands.

This DFB is constructed from two building blocks the first building block is a two channel quincunx filter banks with fan filters (see Figure.5) that divides a 2-D spectrum in to two directions:horizontal and vertical .the second building block is a shearing operator which amount to just reordering of image samples.

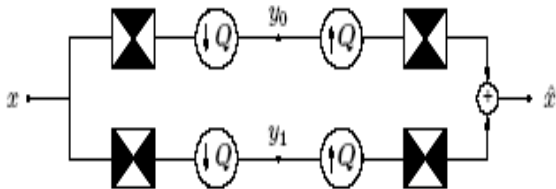


Figure.5 Two-dimensional spectrum partition using quincunx filter banks with fan filters. The black regions represent the ideal frequency supports of each filter. Q is a quincunx sampling matrix.

The following figure shows an application of shearing operator where a -45° edge becomes a vertical edge.



Figure.6 Example of shearing operation that is used like a rotation operation for DFB decomposition. (a) The "cameraman" image. (b) The "cameraman" image after a shearing operation.

By adding a pair of shearing operators and its inverse (unshearing) to before and after, respectively, a two

channel filter bank in Figure.5 we obtained a different directional frequency partition while maintaining perfect reconstruction. Thus the key in, the DFB is to use an approximate combination of shearing operators together with to direction partition of quincunx filter banks at each node in a binary tree structured filter bank, to obtain the desired 2-D spectrum division as shown in the Figuer.4 Using multirate identities ,it is instructive to view an l-level tree structured DFB equivalently as a 2^l parallel channel filter bank with equivalent filters and overall sampling matrices as shown in the Figure.7.Denote these equivalent (directional) synthesis filters as $D_k^{(l)}, 0 \leq k < 2^l$ which correspond to the sub bands indexed as Figure.4.

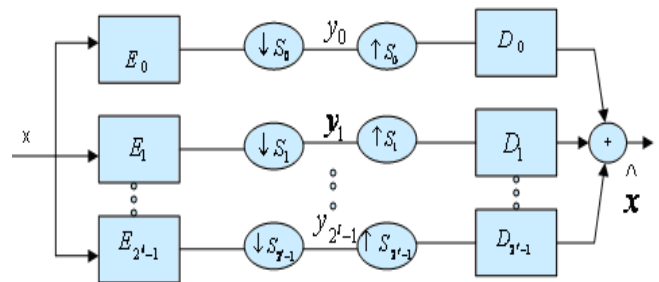


Figure.7 The multichannel view of an l-level tree-structured directional filter bank.The corresponding overall sampling matrices were shown to have following diagonal forms

$$S_k^{(l)} = \begin{cases} \text{diag}(2^{l-1}, 2) & \text{for } 0 \leq k < 2^{l-1} \\ \text{diag}(2, 2^{l-1}) & \text{for } 2^{l-1} \leq k < 2^l \end{cases} \quad (3)$$

Which means sampling is separable. The two sets correspond to the mostly horizontal and mostly vertical sets of directions, respectively. From the equivalent parallel view of the DFB, we see that the family

$$\{d_k^{(l)}[n - S_k^{(l)}m]\} \quad 0 \leq k < 2^l, m \in \mathbb{Z}^2 \quad (4)$$

Obtained by translating the impulse responses of the equivalent synthesis filters $D_k^{(l)}$ over the sampling lattices by

$S_k^{(l)}$ provides a basis for discrete signals in $l^2(\mathbb{Z}^2)$.This basis exhibits both directional and localization properties.

We choose '9-7'biorthogonal filters because they have been shown to provide the best results fir images, partly because they are linear phase and are close to bring orthogonal. In the DFB stage we use the '23-45' ('pkva')biorthogonal quincunx filters designed by Phoong, Kim, Vidyanathan, and Ansari and modulated them to obtain the biorthogonal fan filters. Apart from also being linear phase and nearly orthogonal, these fan filters are closed to having the ideal frequency response and thus can approximate the directional vanishing moment condition.

Multiscale and directional decomposition: The discrete contourlet transform

Combining the Laplacian pyramid and the directional filter bank, we are now ready to describe their combination into a double filter bank structure .since the directional filter bank (DFB) was designed to capture the high frequency (representing directionality)of the input image , the low frequency content is poorly handled .in fact, with the frequency partition shown in Figure.4.,low frequency would leak into several directional sub bands ,hence the DFB alone does not provide a sparse representation for images. This fact provides another reason to combine the DFB with a multiscale decomposition, where low frequencies of input image are removed before applying the DFB.Figure.3.2.7. shows multiscale and directional decomposition using a combination of a Laplacian pyramid (LP) and a directional filter bank (DFB).band pass images from the LP are fed into a DFB so that directional information can be captured .The scheme can be iterated on the coarse image .The combined result is a doubled iterated filter bank structure, named contourlet filter bank, which decomposes images into directional sub bands at multiple scales.

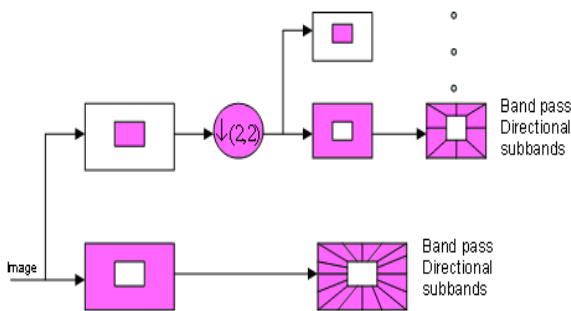


Figure.8. The contourlet filter bank: first, a multiscale decomposition into octave bands by the Laplacian pyramid is computed, and then a directional filter bank is applied to each band pass channel.

Specifically ,let $a_0[n]$ be the input image .the output after the LP stage is J band pass images $b_j[n]$, $j = 1, 2, 3, \dots, J$ (in the fine –to–coarse order) and a low pass image $a_j[n]$.that means , the j -th level of the LP decomposes the image $a_{j-1}[n]$ into a coarser image $a_j[n]$ and a detail image $b_j[n]$ is further decomposed by an l_j level DFB into 2^{l_j} ,Band pass directional image

$$c_{j,k}^{(l_j)}[n], k = 0, 1, 2, \dots, 2^{l_j} - 1.$$

Contourlets and directional multiresolution Analysis:

As for the wavelet filter bank , the contourlet filter bank [9] has an associated continuous –domain expansion in $L_2(R^2)$ using the contourlet functions .The new elements in the frame work are multidirection and it’s combination with multi scale .For simplicity, we will only consider the case with orthogonal filters , which leads tight frames .

We start with the multiresolution analysis for the LP ,which is similar to the one for wavelets .suppose that the LP in the contourlet filter bank uses orthogonal filters by 2 in each dimension (that means $M = \text{diag} (2,2)$ in figure 2).Under certain regularity conditions ,the low pass synthesis filter G in the iterated LP uniquely defines a unique scaling function $\phi(t) \in L_2(R^2)$ that satisfies the following two scale equation

$$\phi(t) = 2 \sum_{n \in Z^2} g[n] \phi(2t - n) \tag{5}$$

$$\text{Let } \phi_{j,n} = 2^{-j} \phi\left(\frac{t - 2^j n}{2^j}\right), j \in Z, n \in Z^2$$

Then the family $\{\phi_{j,n}\}_{n \in Z^2}$ is an orthonormal basis for an approximation subspace V_j at the scale 2^j . further more $\{V_j\}_{j \in Z}$ provide a sequence of multiresolution nested subspaces $\dots V_2 \subset V_1 \subset V_0 \subset V_{-1} \subset V_{-2} \dots$, where V_j is associated with uniform grid of intervals $2^j \times 2^j$ that characterizes image approximation at scale 2^j . the difference images in the LP contain the detail necessary to increase the resolution between two consecutive approximation subspaces .Therefore ,the difference image live in a subspace W_j that is the orthogonal compliment of V_j in V_{j-1} , or $V_{j-1} = V_j \oplus W_j$. In “Framing pyramids “ we show that the LP can be considered as an over sampled filter bank where each polyphase component of difference image $b[n]$ in figure 2 ,together with the coarse image $a[n]$, comes from a separate filter bank channel with the same sampling matrix $\text{diag} (2,2)$.Let $F_i(Z)$, $0 \leq i \leq 3$ be synthesis filters for these polyphase components .these are high pass filters .As for wavelets ,we associated with each of these filters a continuous functions $\varphi^{(i)}(t)$ where

$$\varphi^{(i)}(t) = 2 \sum_{n \in Z^2} f_i[n] \phi(2t - n) \tag{6}$$

using the above equation we can say that if

$$\varphi_{j,n}^{(i)}(t) = 2^{-j} \varphi^{(i)}\left(\frac{t - 2^j n}{2^j}\right), j \in Z, n \in Z^2 \tag{7}$$

Then for scale 2^j , $\{\varphi_{j,n}^{(i)}\}_{0 \leq i \leq 3, n \in Z^2}$ is a tight frame

for W_j . For all scales,

$\{\varphi_{j,n}^{(i)}\}_{j \in Z, 0 \leq i \leq 3, n \in Z^2}$ is a tight frame for L_2

(R^2) .in both cases the frame bounds are equals to 1.

B.Multidirection

In the iterated contourlet filter bank, the discrete basis of the DFB can be regarded as a change of basis for the continuous domain subspaces for the multiscale analysis in the last section. Suppose that the DFB 's in the contourlet bank uses orthogonal filters. Although in the contourlet transform the DFB is applied to difference images or the detail subspaces W_j we first show what happens when the DFB is applied to the approximation subspaces V_j . Suppose that

$\left\{ \rho_{j,k,n}^{(l)} \right\}_{n \in \mathbb{Z}^2}$ is an orthonormal of subspace $V_{j,k}^{(l)}$. to

increase the directional resolution, an extra level of decomposition by pair of orthogonal filter is applied to the channel represented by $d_k^{(l)}$ that leads to two channels with equivalent filters $d_{2k}^{(l+1)}$ and $d_{2k+1}^{(l+1)}$. This transform the

orthonormal basis $\left\{ \rho_{j,k,n}^{(l)} \right\}_{n \in \mathbb{Z}^2}$. Each of

These families generate subspace with finer directional resolution.

So we can say that if $\rho_{j,k,n}^{(l)}(t) = \sum_{m \in \mathbb{Z}^2} d_k^{(l)}[m - S_k^{(l)}n] \phi_{j,m}(t)$ for arbitrary but

finite l. Then the family $\left\{ \rho_{j,k,n}^{(l)} \right\}_{n \in \mathbb{Z}^2}$ is an orthonormal

basis of directional subspace $V_{j,k}^{(l)}$ for each $k=0, 1, 2, \dots, 2^l-1$. Furthermore

$$V_{j,k}^{(l)} = V_{j,2k}^{(l+1)} \oplus V_{j,2k+1}^{(l+1)} \quad (8)$$

$$V_{j,k}^{(l)} \perp V_{j,k'}^{(l)} \text{ for } k \neq k' \text{ and } \quad (9)$$

$$V_j = \bigoplus_{k=0}^{2^l-1} V_{j,k}^{(l)} \quad (10)$$

III.II Image Denoising using contourlet transform

The improvement in approximation by contourlets based on keeping the significant coefficients will directly lead to improvement in applications, including compression, denoising, and feature extraction. As an example, for image denoising, random noise will generate significant wavelet coefficients just like true edges, but is less likely to generate significant contourlet coefficients. Consequently, a simple thresholding scheme applied on the contourlet transform is more effective in removing the noise than it is for the wavelet transform.

The algorithm is

1. Choose the appropriate filters for pyramidal and directional decomposition.
2. Choose the no of levels in the pyramid levels of pyramid.

3. Choose the appropriate values for threshold (Th) and for noise level (rho).

4. Generate noisy image.

5. Perform multiscale and multidirectional decomposition on the noisy image.

6. Calculate the contourlet threshold by using the given threshold.

7. Apply thresholding for noisy coefficients.

8. Perform multistage and multidirectional reconstruction to reconstruct the denoised image.

9. Calculate the PSNR value to compare it with wavelet PSNR.

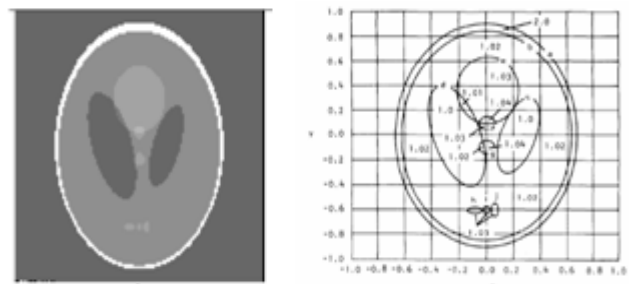
The contourlet transform is shown to be more effective in recovering smooth contours, both visually as well as in PSNR than wavelets.

IV. IMPLEMENTATION AND RESULTS

Implementation

The well-known Shepp-Logan 'head phantom' of size 256X256, is taken as test images. The number of angles is 256 and the number of projections is 256. It is shown in Fig 9.

The image in Fig. 9 is composed of 10 ellipses, as illustrated in Fig 10. The parameters of these ellipses are given in Table-1



Centre	Major axis	Minor axis	Rotation angle	Refractive index
(0, 0)	0.92	0.69	90	2.0
(0, -0.0184)	0.874	0.6624	90	-0.98
(0.22, 0)	0.31	0.11	72	-0.02
(-0.22, 0)	0.41	0.16	108	-0.02
(0, 0.35)	0.25	0.21	90	0.01
(0, 0.1)	0.046	0.046	0	0.01
(0, -0.1)	0.046	0.046	0	0.01
(-0.08, -0.605)	0.046	0.023	0	0.01
(0, -0.605)	0.023	0.023	0	0.01
(0.06, -0.605)	0.046	0.023	90	0.01

Table-1: Summary of parameters for tomography simulation

The three types of noise, Gaussian of different means and different variances, multiplicative noise (speckle) of different variances, salt & pepper of different densities are added to the test image. The algorithms of FBP, wavelets and contourlets are implemented in

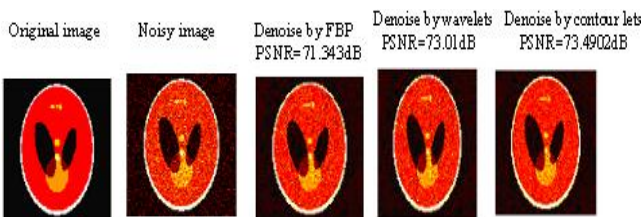
MATLAB. The performance of these algorithms are compared by using peak signal to noise ratio(PSNR) value. Contourlets are shown to be superior compared to wavelets in capturing fine contours. In addition to there is significant gain in PSNR for contourlets.

Results

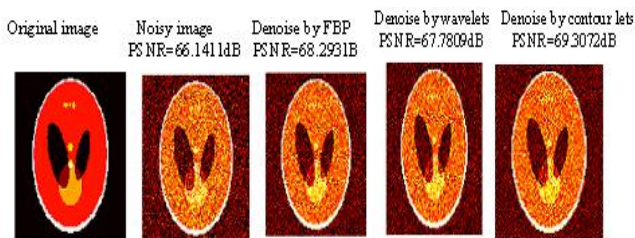
The results of all the 3 algorithms FBP, wavelets, contourlets when those are applied to denoise the head

photon image having different properties of noises are given below.

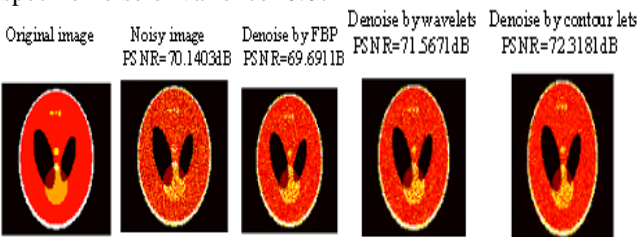
Gaussian noise of mean=0, variance=0.01



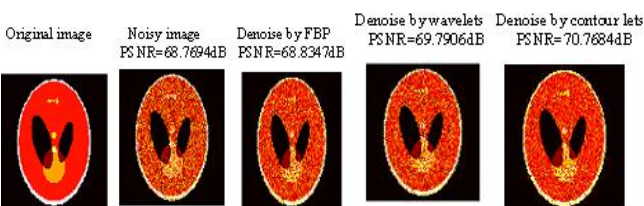
Gaussian noise of mean=0.1, variance=0.02



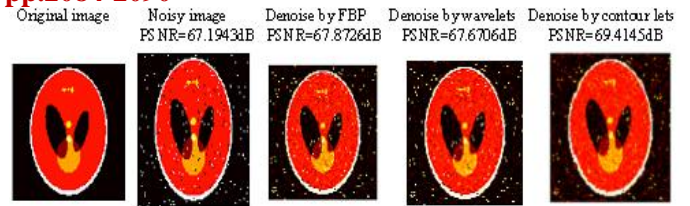
speckle noise of variance=0.07



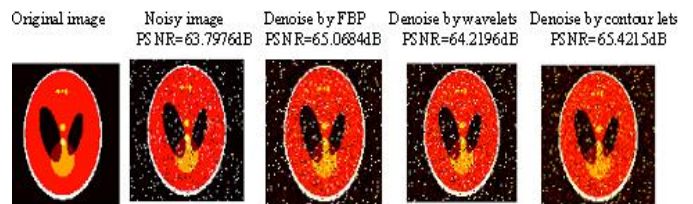
speckle noise of variance=0.1



Salt & pepper noise of density =0.03



Salt pepper noise of density =0.07



Performance tables

Table-2 Gaussian noise

Property of noise	Noisy image PSNR(dB)	Denoise by FBP PSNR(dB)	Denoise by wavelet PSNR(dB)	Denoise by contourlet PSNR(dB)
Mean=0 Variance=0.01	70.0445	71.343	73.0108	73.4902
Mean=0.1 Variance=0.01	68.7179	69.7163	71.7827	72.8085
Mean=0.1 Variance=0.02	66.141	68.293	67.7809	69.3072

Table-3 Speckle noise

Property of noise	Noisy image PSNR(dB)	Denoise by FBP PSNR(dB)	Denoise by wavelets PSNR(dB)	Denoise by contourlets PSNR(dB)
Variance=0.07	70.1403	69.6911	71.0671	72.3181
Variance=0.1	68.7694	68.8347	69.7906	70.7684
Variance=0.15	66.9267	67.6944	67.5615	68.4486

Table-4 Salt & pepper noise

Property of noise	Noisy image PSNR(dB)	Denoise by FBP PSNR(dB)	Denoise by wavelets PSNR(dB)	Denoise by contourlets PSNR(dB)
Density=0.03	67.1943	67.8726	67.6706	69.4145
Density=0.05	65.0876	66.2351	65.539	67.0255
Density=0.07	63.7976	65.0684	64.219	65.4215

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V.CONCLUSIONS

In this paper, how a tomographic noisy image is denoised by using contourlet transform is presented. Using recent results from harmonic analysis and vision, we first identified two key features of contourlet transform that improves over the separable 2-D wavelet transform, namely directionality and anisotropy. Based on this observation, we developed a new filter bank structure, the contourlet filter

bank, that can provide a flexible multiscale and directional decomposition for images. The developed discrete filter bank has a precise connection with the associated continuous-domain contourlet expansion. This connection is defined via a directional multiresolution analysis that provides successive refinements at both spatial and directional resolution. Along with the contourlet algorithm, wavelet and FBP algorithms and their advantages and limitations are presented in this paper. The results show that wavelets are superior to FBP. The contourlet transform is shown to be more effective than wavelets in recovering smooth contours, both visually as well as in PSNR.

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